*Solve coin tossing problems using transition matrix of Markov Chains*

**HH: Expected number of tossing a fair coin until you get two Heads in a row?**

**Head, Head**

|  |  |  |  |
| --- | --- | --- | --- |
|  | State 0 | State 1 | State 2 |
| State 0 | 0.5 | 0.5 | 0 |
| State 1 | 0.5 | 0 | 0.5 |
| State 2 | 0 | 0 | 1 |

*ϕ(2) = 0*

ϕ(0)=0.5∗(1+ϕ(0))+0.5∗(1+ϕ(1))+0∗(1+ϕ(2))

ϕ(1)=0.5∗(1+ϕ(0))+0∗(1+ϕ(1))+0.5∗(1+ϕ(2))

**HH: ϕ(0) = 6**

**TT: ϕ(0) = 6**

**HT: Expected number of tossing a fair coin until you get Head, Tail successively?**

**Head, Tail**

|  |  |  |  |
| --- | --- | --- | --- |
|  | State 0 | State 1 | State 2 |
| State 0 | 0.5 | 0.5 | 0 |
| State 1 | 0 | 0.5 | 0.5 |
| State 2 | 0 | 0 | 1 |

*ϕ(2) = 0*

ϕ(0)=0.5∗(1+ϕ(0))+0.5∗(1+ϕ(1))+0∗(1+ϕ(2))

ϕ(1)=0∗(1+ϕ(0))+0.5∗(1+ϕ(1))+0.5∗(1+ϕ(2))

**HT: ϕ(0) = 4**

**TH: ϕ(0) = 4**

**HHH: Expected number of coin flips until you get three Heads in a row?**

*Apply transition matrix of Markov chains to solve this problem,*

ϕ(0): state at start, ϕ(1), ϕ(2), ϕ(3): state of getting 1, 2, 3 Head tosses

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 |
| 0 | 0.5 | 0.5 | 0 | 0 |
| 1 | 0.5 | 0 | 0.5 | 0 |
| 2 | 0.5 | 0 | 0 | 0.5 |
| 3 | 0 | 0 | 0 | 1 |

ϕ(0) = 0.5∗(1+ϕ(0))+0.5∗(1+ϕ(1))+0∗(1+ϕ(2))+0\*(1+ϕ(3))

ϕ(1) = 0.5∗(1+ϕ(0))+0∗(1+ϕ(1))+0.5∗(1+ϕ(2))+0\*(1+ϕ(3))

ϕ(2) = 0.5∗(1+ϕ(0))+0∗(1+ϕ(1))+0∗ϕ(2) +0.5\*(1+ϕ(3))

*ϕ(3) = 0*

**HHH: ϕ(0) = 14**

**H^n: expected number of coin flips until you get n heads in a row?**

**H^n: ϕ(0) = =2\*( 1)**